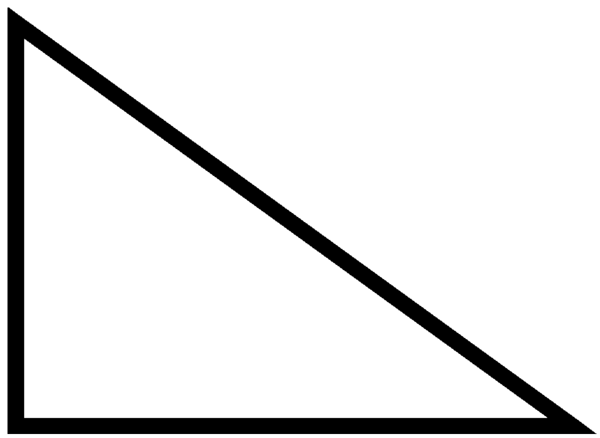
THE PYTHAGOREAN THEOREM



$$a^2 + b^2 = c^2$$

CFE 3285V

OPEN CAPTIONED
ALLIED VIDEO CORPORTATION
1993
Grade Levels: 9-12
14 minutes

DESCRIPTION

Reviews the definition of a right triangle before a brief history of the development of the Pythagorean theorem. Shows how to use the theorem to solve for the hypotenuse (*c*) and each of the two sides (*a* and *b*). Briefly presents applications of the theorem.

INSTRUCTIONAL GOALS

- To review the right triangle and its parts.
- To re-create the development of the Pythagorean theorem.
- To rewrite the Pythagorean theorem to solve for *a* and *b*.
- To show how to use the Pythagorean theorem.
- To present practical applications of the Pythagorean theorem.

BEFORE SHOWING

1. Read the CAPTION SCRIPT to determine unfamiliar vocabulary and language concepts.
2. Using the calculator, practice finding the square and square roots of numbers.
3. Make a time line from 585 BC to the present. Point out the period of time in which Pythagoras lived.

DURING SHOWING

1. View the video more than once, with one showing uninterrupted.
2. Pause at the scene showing the bust of Pythagoras. His discovery has made him famous for 2600 years. Determine how that number was derived.
3. Pause at the scene showing the substitution of numerical values for *a* and *b*.
    a. Substitute other values for *a* and *b* and calculate *c*.
    b. Explain that the answer for *c* is usually a radical expression, not an integer.

4. Pause at the scenes showing how to rewrite the theorem to solve for *a* and *b*.
    a. Using the blackboard, visually explain the steps in more detail.
    b. Substitute other values for *c* and *a* (or *b*) to find the remaining side.
5. Pause after the scene showing the wall and ladder application. Re-create that demonstration in the classroom and calculate the value for *b*.

AFTER SHOWING

1. Use a computer generated graphic to construct right triangles. Give values for two of the sides and solve for the third side.
2. Cut a 3" x 5" index card into three right triangles.
    a. Use a ruler to measure the lengths of the sides.
    b. Design a chart with these column headings: Short side, Long side, Hypotenuse, $(\text{Short side})^2$, $(\text{Long side})^2$, and $(\text{Hypotenuse})^2$.
    c. Record the measurements in the chart and look for a pattern.
3. Research and report on Pythagoras and the secret math society to which he belonged.
4. Records of this theorem's use in northern Africa predate Pythagoras by hundreds of years. Report on this topic.
5. According to *The Guinness Book of World Records*, the Pythagorean theorem has more published proofs than any other theorem.
    a. Identify and explain one or more of these proofs.
    b. One resource for this is the Ask Dr. Math website: http://forum.swarthmore.edu/dr.math/
6. Find two different Pythagorean triples that have 24 as one of the numbers.
7. Design a work sheet of practical applications of the Pythagorean theorem.
8. Prove the converse of the Pythagorean theorem.

9. List some practical applications of the converse of the Pythagorean theorem.

10. Report on the development of the square root. Determine what methods were used to calculate square roots in ancient times.

11. Design a work sheet with numerical values for *a*, *b*, and *c*. Determine which are values for a right triangle.

WEBSITES

Explore the Internet to discover sites related to this topic. Check the CFV website for related information (http://www.cfv.org).

CAPTION SCRIPT

Following are the captions as they appear on the video. Teachers are encouraged to read the script prior to viewing the video for pertinent vocabulary, to discover language patterns within the captions, or to determine content for introduction or review. Enlarged copies may be given to students as a language exercise.

[piano playing "Chopsticks"]

(male narrator)
Welcome to the assistant
professor's

educational video series.

Today, we are studying:

The first lesson defines
the Pythagorean Theorem.

The second lesson will
solve some example problems,

and explain how to use
the theorem.

Before we begin,

let's review the definition
of a right triangle.

A right triangle has one
90 degree, or right, angle.

Usually, the sides of a
right triangle are named:

Side "c" is called
the *hypotenuse*.

The hypotenuse is always the
side opposite the right angle.

Sides "a" and "b" are simply
called sides.

The sides meet to form
the 90 degree angle.

This object is a natural,
perfect right triangle.

Its sides always measure
3 units, 4 units,

and its hypotenuse,
5 units.

The ancient Greeks discovered
this natural right triangle,

the *builder's triangle*.

They used it to construct
houses, buildings, and temples.

Using the builder's triangle,

they could insure that their
corners, joints, and vertices

were perfect 90 degree angles.

In about 600 B.C.,
on the Greek island of Samos,

there lived a man
named Pythagoras.

Pythagoras was a mathematician
and a philosopher.

Pythagoras was fascinated
by geometry,

and he was interested
in triangles.

He noticed something interesting

about the shape and measurements
of the builder's triangle.

He made a square out of
the length of each side

of the builder's triangle.

Pythagoras took the square area

formed by the side measuring
3 units,

and placed it in the square
made from the hypotenuse.

Then, Pythagoras took
the area of the square

formed by the side measuring
4 units,

and placed it inside the square
made from the hypotenuse.

The total area of the squares

made from the sides was exactly
the same as the area

of the square made
from the hypotenuse.

Pythagoras thought this result
was interesting.

He tried the same idea
on other right triangles.

He tried it on thousands
of triangles.

He found the same result

on every right triangle
he measured.

So, Pythagoras formulated
his theorem.

For every right triangle,

The sum of the areas formed
by the squares

of the lengths of the sides is
equal to the area of a square

made by the length
of the hypotenuse.

In other words:

This discovery was so important
that it has made Mr. Pythagoras

famous for 2,600 years.

Let's review this important
idea again.

In every right triangle,
the sum of the areas

formed by the squares
of the sides

is equal to the area formed by
the square of the hypotenuse.

Now that we know what the
Pythagorean Theorem is,

let's solve some problems.

If you know the length of two
sides of a right triangle,

you can find the length
of the third side.

We can use the Pythagorean
Theorem to find the length

of the third side.

This right triangle has sides
measuring 3 units and 4 units.

First, substitute the values
into the equation.

3 units squared equals
9 square units.

4 units squared equals
16 square units.

9 plus 16 equals 25.

The value of "c" squared
is 25.

However, we want to solve
for the value of "c"--

not "c" squared.

To solve for "c,"
take the square root

of both sides of the equality.

Taking the square root
of a value

is the reverse process
of squaring a value.

The square root of "c" squared
is "c."

The square root of 25 is 5.

The value of "c," or the length
of the hypotenuse, is 5 units.

We know the value for each side
of this triangle.

That was pretty easy.

The Pythagorean Theorem works.

Pythagoras' theorem
is also versatile.

We can manipulate it
to create a formula

that solves for the length
of the hypotenuse.

For any right triangle whose
sides are labeled "a" and "b,"

and whose hypotenuse is "c,"

the length of "c" is equal
to the square root

of "a" squared
plus "b" squared.

This expression states
that we are solving

for the length of "c,"
the hypotenuse.

We substitute the values
in the formula,

and solve
for the value of "c."

We know how to find the length
of the hypotenuse,

but how can we find side "b?"

For this right triangle, suppose
we know the length of side "a,"

and the length of the
hypotenuse, or side "c."

How do we solve for the length
of side "b?"

Again, we can manipulate
Pythagoras' famous theorem:

To isolate the value of "b,"
we subtract

"a" squared from both sides
of the equality.

This leaves us
with the equality:

To find the value of side "b,"

we take the square root
of each side of the equality.

The square root of "b" squared
is "b."

The value of "b" equals the
square root of the expression:

Let's use this expression to
find side "b" of this triangle.

Side "c" is 4 units.

Side "a" is 3 units.

Using the formula, substitute
the values for "c" and "a."

4 squared equals 16,
and 3 squared equals 9.

16 minus 9 equals 7.

"b" equals the
square root of 7.

Unfortunately, Pythagoras
did not have a calculator,

so he had to do a lot
of arithmetic.

We use the square root function
on a calculator,

and find that the square root
of 7 equals 2.646.

The length of side "b"
is 2.646 units.

Let's move on to another
example.

We know how to find two of
the sides of a right triangle.

How can we find
the third side, "a?"

We can begin with
the original theorem,

but we need to rearrange it
to solve for the value of "a."

To isolate the value of "a,"
subtract the value

of "b" squared from
both sides of the equality.

Since we are solving
for the value of "a,"

take the square root of both
sides of the equality.

The value of side "a" equals
the square root of "c" squared

minus "b" squared.

In this example,
let's suppose that side "c"

is 7 units long and that
side "b" is 5 units long.

Using the formula, substitute
the values in the equation,

and then solve.

7 squared is 49,
and 5 squared is 25.

49 minus 25 equals 24.

The length of side "a"
equals the square root of 24.

Using a calculator,
the length of side "a"

is about 4.899 units.

Let's review what we learned
about the Pythagorean Theorem.

or any right triangle whose
sides are named "a" and "b,"

and whose hypotenuse
is named "c:"

The length of the hypotenuse,
or side "c," is:

The length of side "a" is:

The length of side "b" is:

What's so important
about Pythagoras?

Something must be important
since we remember him

after 3,000 years.

Pythagoras is the first known
person to develop the idea

that the physical world can
be modeled and represented

through pure mathematics.

Pythagoras showed us that
mathematics is a special kind

of language we can use
to develop ideas,

predict outcomes,
and solve problems.

Problems can be solved

mathematically on paper
instead of relying

on tedious experimentation,
or trial and error.

Let's look at a problem
you might solve

while learning about
the Pythagorean Theorem.

How far does this ladder
reach up the wall?

To solve this,
we can build a model.

First, name the length
of the ladder "c."

Use "a" to name the distance
between the wall and the ladder.

And use "b" to name
the length of the wall.

This triangle is a graphic
model of our problem.

Pythagoras gave us a way to
build a mathematical model.

This equation represents
the real wall and ladder

just as the triangle represented
the real wall and ladder.

Applications of the Pythagorean
Theorem are all this simple.

Even in complex
mathematical problems,

this simple application
of the Pythagorean Theorem

is used over and over.

The Pythagorean Theorem
is one of the building blocks

for more complex
mathematical expressions.

It is used to model intricate
problems like putting

satellites in orbit,

constructing bridges,
designing buildings, and so on.

Mathematics shapes and builds
the world around us.

Science, technology,
and industry as we know it

would not exist without
mathematics.

Understanding Pythagoras'
theorem is an important

step toward mastering
mathematics,

and mastering your world.

Good-bye, and good luck.

Funding for purchase
and captioning of this video

was provided by the
U.S. Department of Education:

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